# Analytical expression for long exposure coronagraphic imaging
# -
# First applications

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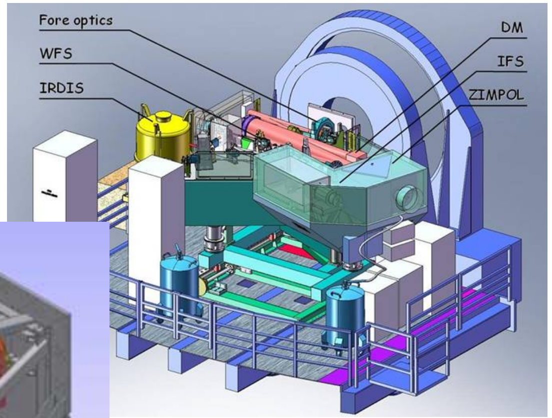
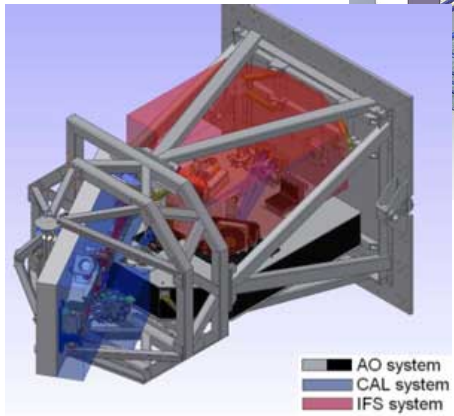
retour sur innovation

# Global context high contrast imaging

- Ground direct imaging of exoplanet
  - SPHERE
  - GPI




- Instrument type
  - Direct imaging / SDI / ADI
  - IFS
  - Polarimetric imaging

- Need for dedicated data processing
  - Quasi-static wavefront control
  - *A posteriori* Deconvolution

- = > Coronagraphic image model accounting for averaged turbulence residuals

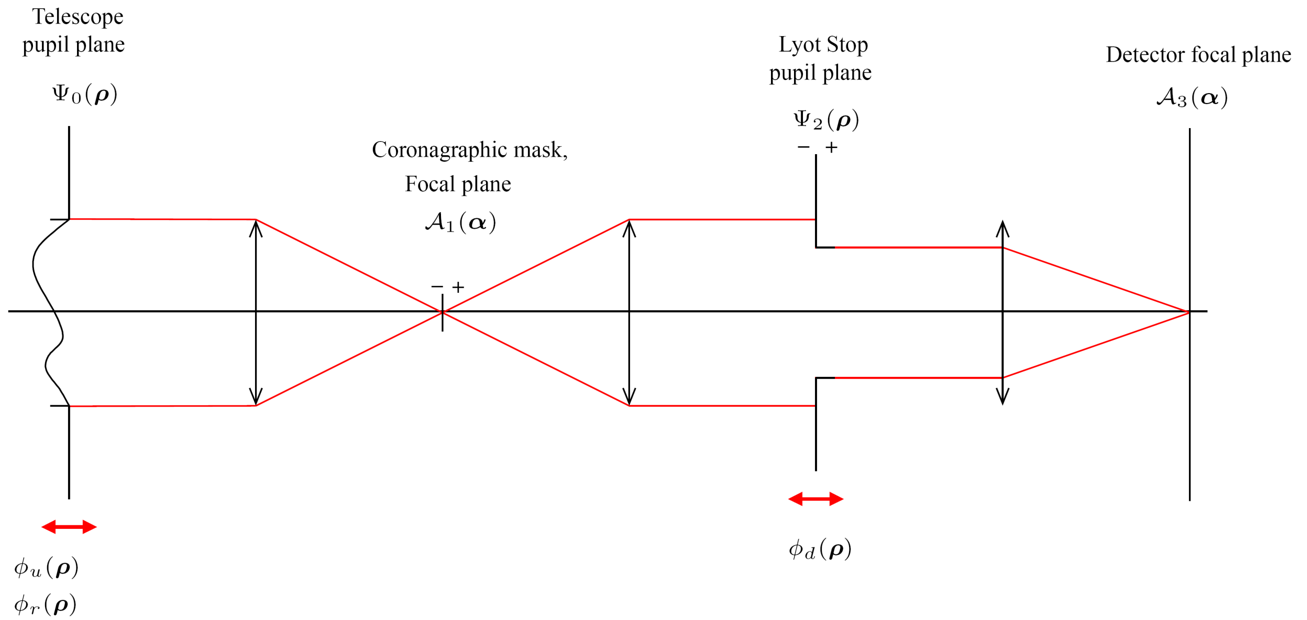
Direction - Conférence

# Outline

- Global context : High contrast imaging
- Analytical expression of coronagraphic imaging
- First applications
- Conclusion / perspectives

# Assumptions

- Coronagraphic scheme :
  - Residual turbulence, time-dependent
  - Static upstream aberrations
  - Perfect coronagraph
  - Static downstream aberrations




- Long-exposure expression without small phase assumption ?





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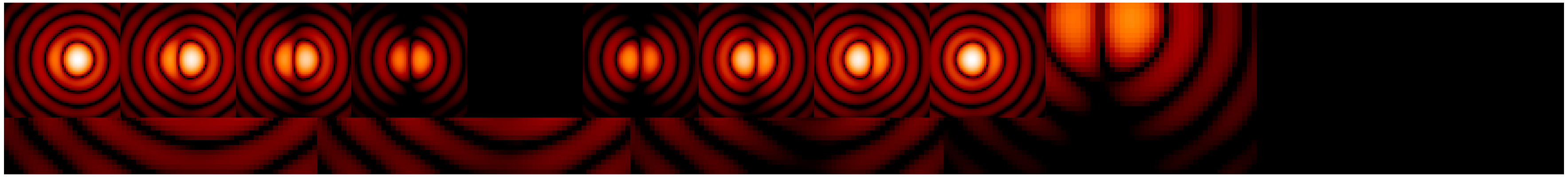
# Definition of perfect coronagraph

- Idealized device subtracting a plane wave in pupil plane, in a proportion that minimizes outgoing energy

$$\mathcal{A}_1^+(\boldsymbol{\alpha}, t) = \mathcal{A}_1^-(\boldsymbol{\alpha}, t) - \eta(t)\mathrm{FT}^{-1}(\mathcal{P}_u(\boldsymbol{\rho}))$$

$$\eta_0(t) = \underset{\eta(t)}{\arg\min} \left|\left|\mathcal{A}_1^-(\boldsymbol{\alpha}, t) - \eta(t)\mathrm{FT}^{-1}(\mathcal{P}_u(\boldsymbol{\rho}))\right|\right|^2$$

$$\eta_0(t) = \langle \Psi_0(\boldsymbol{\rho}, t) | \mathcal{P}_u(\boldsymbol{\rho}) \rangle$$



Post-coronagraphic PSF with Tip from –1.6 to 1.6 rad

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# Validation by simulations

Sauvage et al JOSA A, Vol. 27 Issue 11, Novembre 2010

$$h_c\left(\phi_u, \phi_d, D_\phi\right)$$

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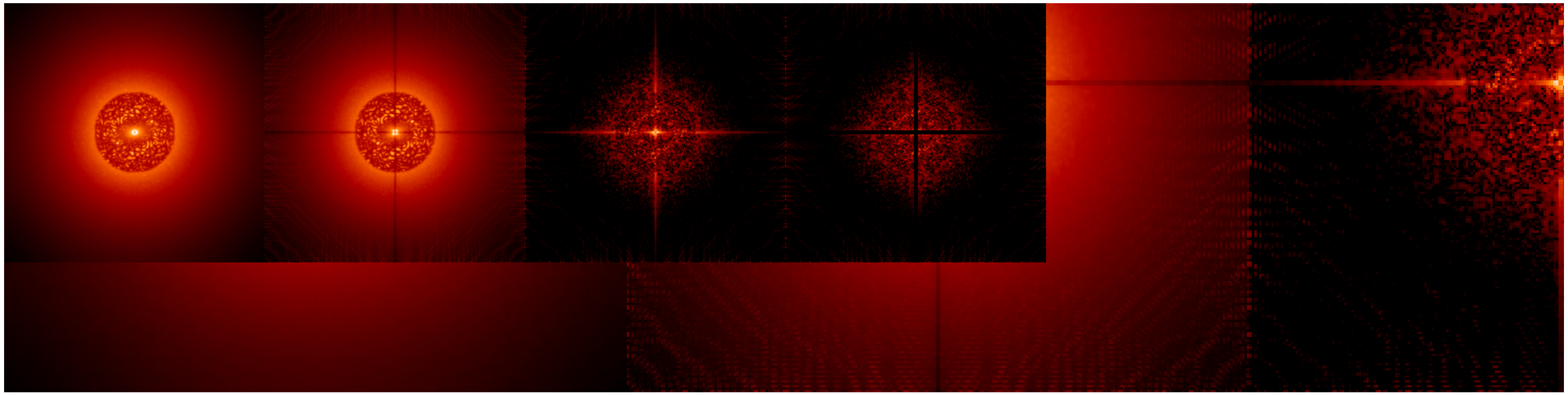
$$h_c\left(\phi_u, \phi_d, D_\phi\right)$$

- Comparison to empirical sum of short exposures
  - $\phi_u$ = 35nm, $\phi_d$ = 100nm
  - Residual AO turbulent wavefront, Strehl Ratio = 90%
  - Four Quadrant Phase Mask coronagraph



$h_c\left(D_\phi, \phi_u, \phi_d\right)$ $\left\langle h\left(\phi_t, \phi_u, \phi_d\right)\right\rangle_t$

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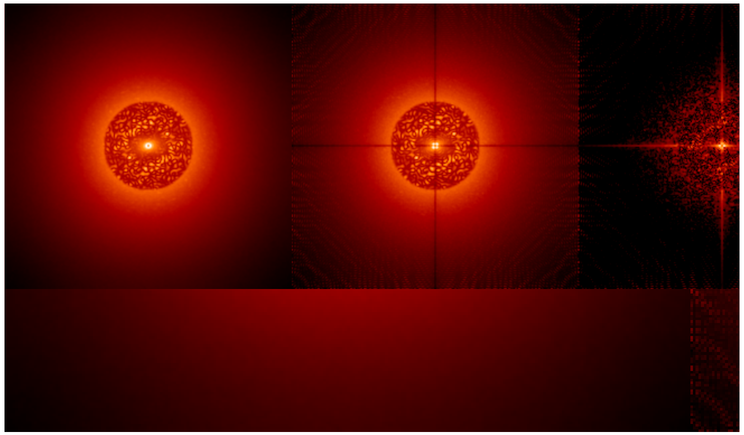

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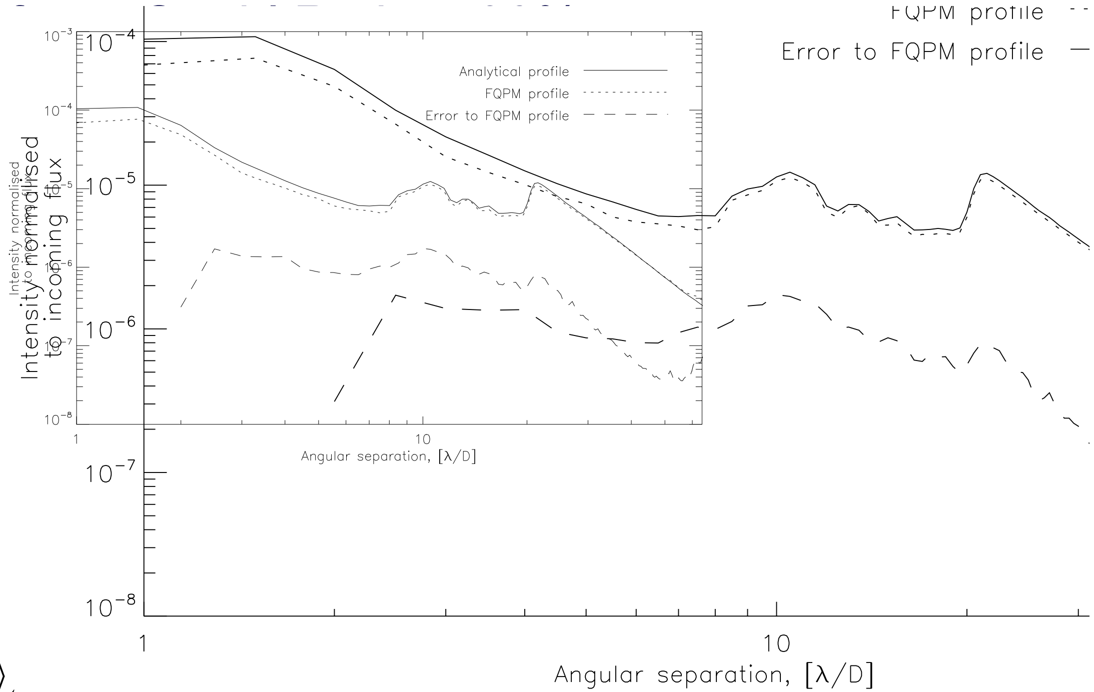


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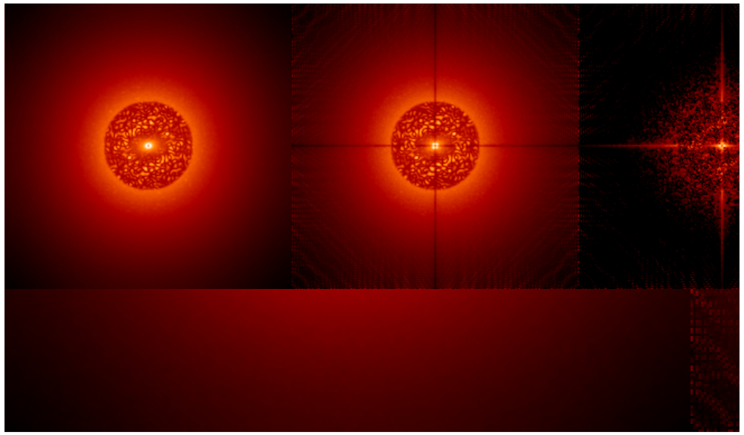

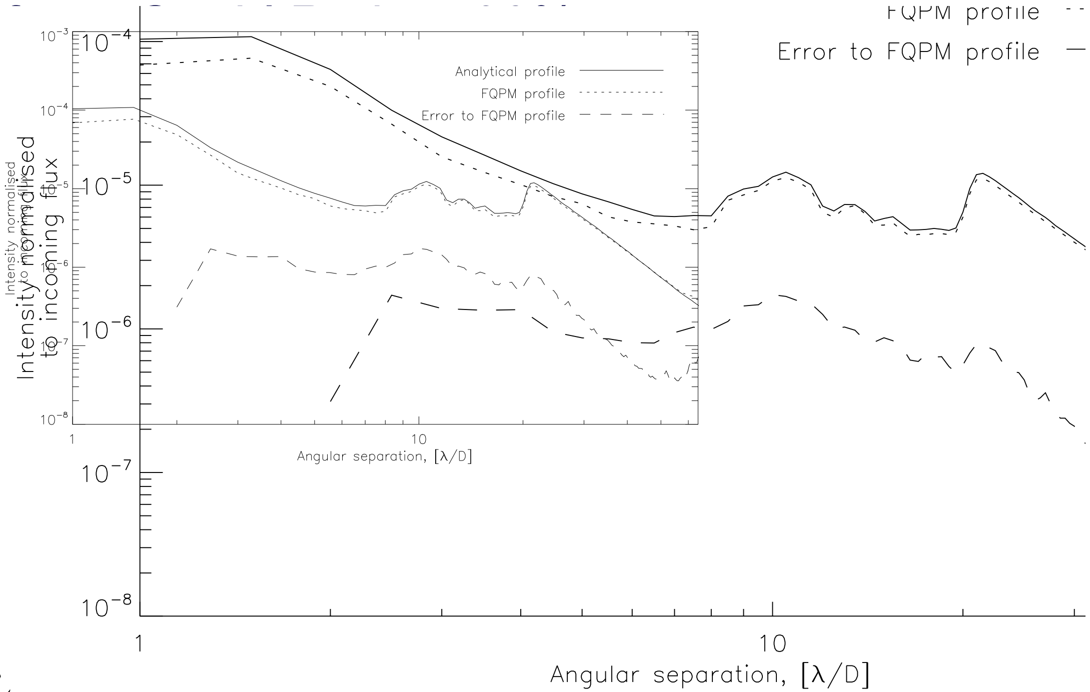


$h_c(D_\phi, \phi_u, \phi_d)$ $\langle h(\phi_t, \phi_u, \phi_d)\rangle_t$

- Model validated wrt summation of short exposures
- Compatibility with 4QPM



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# Application 1/3 : simple deconvolution

- Image formation including star environment

$$i_c = \alpha\, h_c(\phi_u, \phi_d, D_\phi) + o * h_{nc}(\phi_u, \phi_d, D_\phi) + n$$

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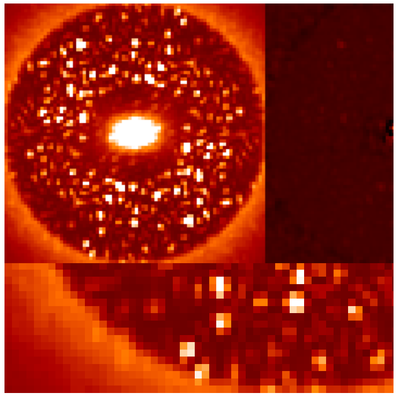
- Image formation including star environment

$$i_c = \alpha\, h_c(\phi_u, \phi_d, D_\phi) + o * h_{nc}(\phi_u, \phi_d, D_\phi) + n$$

- Assume observation parameters almost known $(\hat{\phi}_u, \hat{\phi}_d, \hat{D}_\phi)$
- Companion, contrast $3.10^4$



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# Application 1/3 : simple deconvolution

- Image formation including star environment

$$i_c = \alpha\, h_c(\phi_u, \phi_d, D_\phi) + o \times h_{nc}(\phi_u, \phi_d, D_\phi) + n$$

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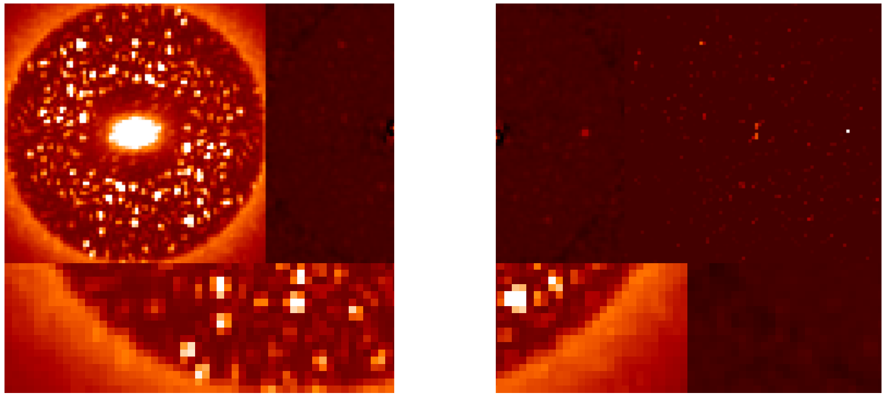

Subtraction of the star image model

$$\hat{\alpha}\, h_c(\hat{\phi}_u, \hat{\phi}_d, \hat{D}_\phi)$$

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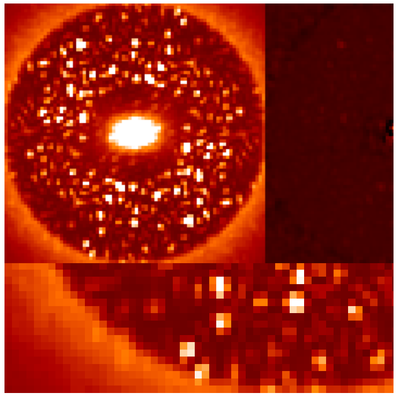

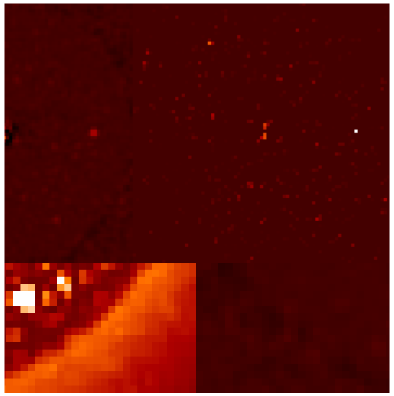

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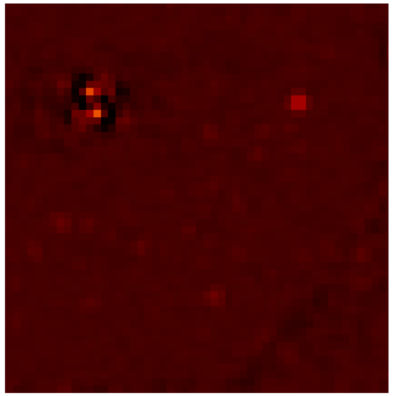

Classical deconvolution (MISTRAL)

$h_{nc}(\hat{\phi}_u, \hat{\phi}_d, \hat{D}_\phi)$

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# Classical phase diversity

- Classical phase diversity :
  - 1 focus and 1 diversified image

$$\mathbf{i}_{foc} = \alpha h_{foc}(\phi) = \alpha \left| \mathrm{FT}\left(P e^{j\phi}\right) \right|^2$$

$$\mathbf{i}_{div} = \alpha h_{div}(\phi) = \alpha \left| \mathrm{FT}\left(P e^{j(\phi + \phi_{div})}\right) \right|^2$$

# Classical phase diversity

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  - MAP criterion minimisation leads to phase $\phi$ estimation

$$\mathrm{J}(\phi) = \left\|\mathbf{i}_{foc} - \alpha h_{foc}(\phi)\right\|^2 + \left\|\mathbf{i}_{div} - \alpha h_{div}(\phi)\right\|^2$$

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# Coronagraphic phase diversity

- Coronagraphic phase diversity :
  - Point like source
  - Static aberrations upstream and downstream of coronagraphic mask
  - 1 focus and 1 diversified image

$$\mathbf{i}^c_{foc} = \alpha h^c_{foc}\left(\phi_u, \phi_d, D_\phi\right)$$
$$\mathbf{i}^c_{div} = \alpha h^c_{foc}\left(\phi_u + \phi_{div}, \phi_d, D_\phi\right)$$



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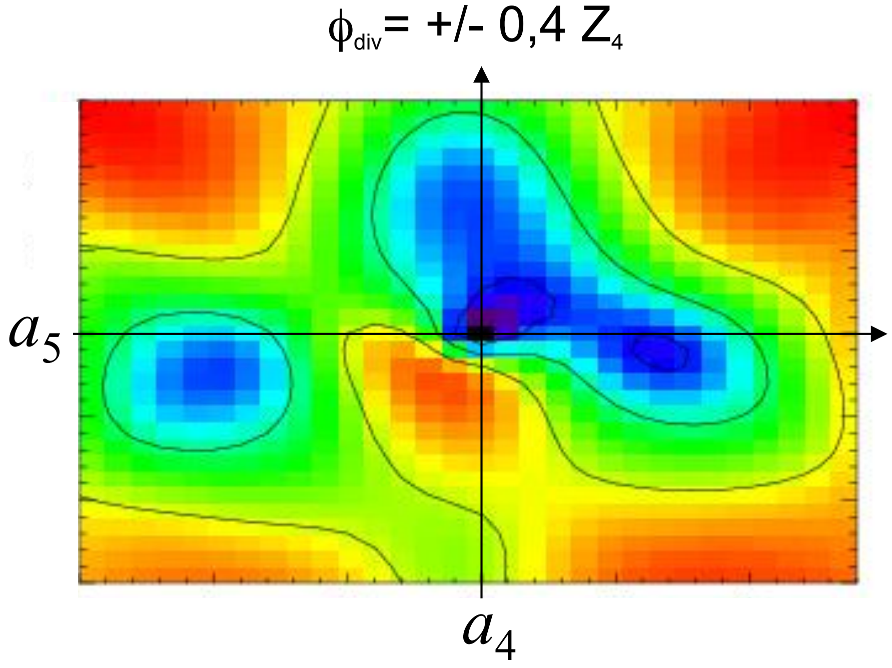
  - MAP criterion minimisation leads to phase $\phi$ estimation

$$\mathrm{J}(\phi_u, \phi_d, \alpha) = \left\| \mathbf{i}^c_{foc} - \alpha h^c_{foc}(\phi_u, \phi_d, D_\phi) \right\|^2 + \left\| \mathbf{i}^c_{div} - \alpha h^c_{foc}(\phi_u + \phi_{div}, \phi_d, D_\phi) \right\|^2$$

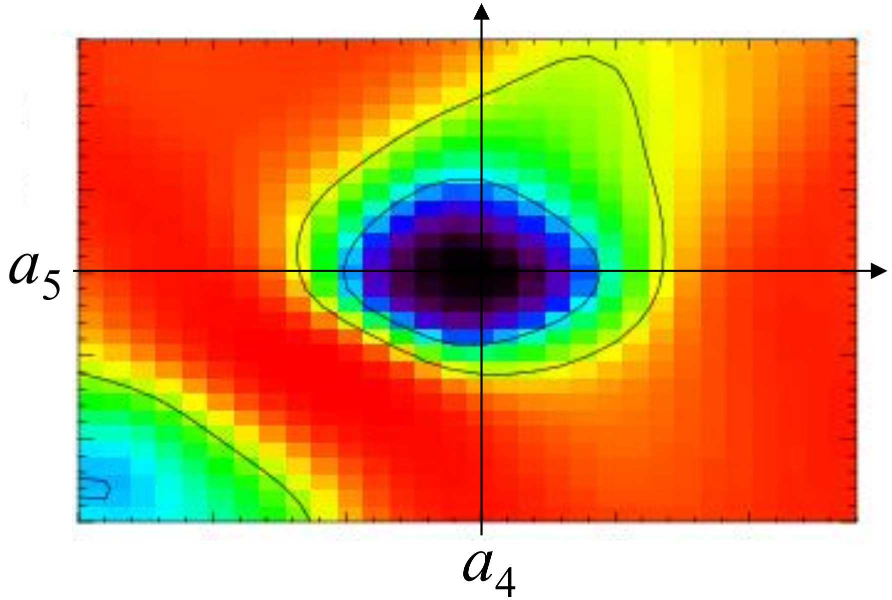
# Choice of phase diversity




Criterion map for an estimated phase $\phi = a_4\mathbf{Z}_4 + a_5\mathbf{Z}_5$

# Choice of phase diversity

$\phi_{div} = 0.8\ Z_4 + 0.8\ Z_5$

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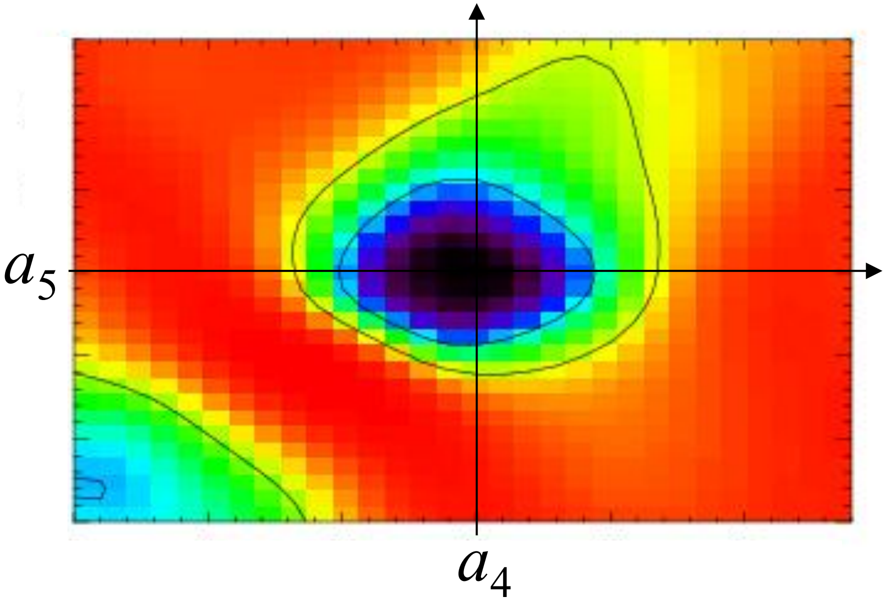
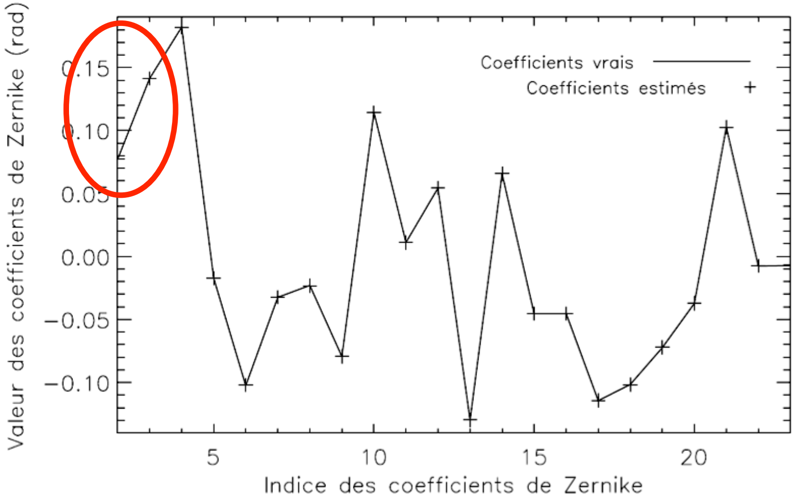


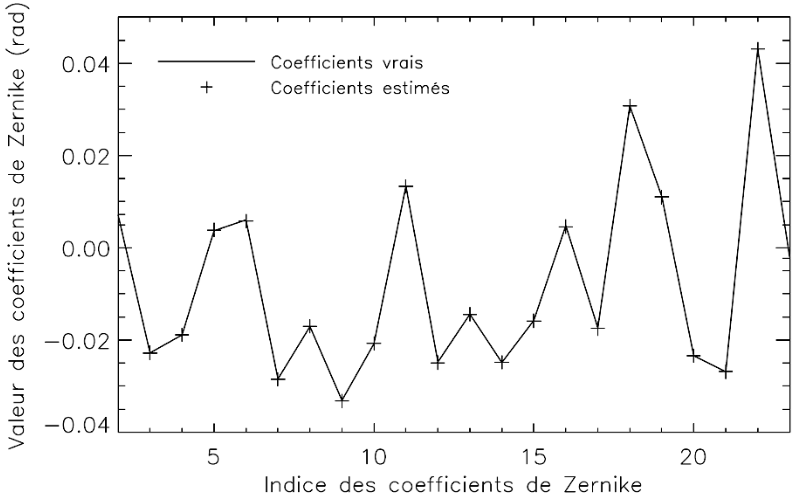

Criterion map for an estimated phase $\phi = a_4\mathbf{Z}_4 + a_5\mathbf{Z}_5$

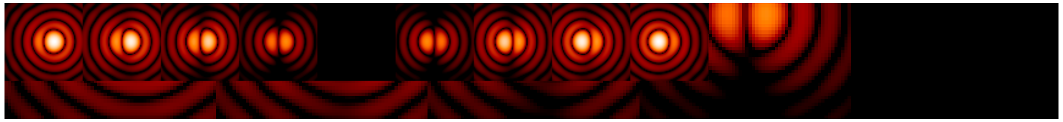
Selection of cross-diversity $\phi_{div} = 0.8\mathbf{Z}_4 + 0.8\mathbf{Z}_5$

# Simulation results on static aberrations

- Estimation of 20+20 Zernike modes, including Tip-Tilt
- Static aberrations representative of SPHERE constraints
- Joint estimation of upstream and downstream aberrations
- Nanometric accuracy



- Downstream aberrations can be jointly estimated with upstream abs.

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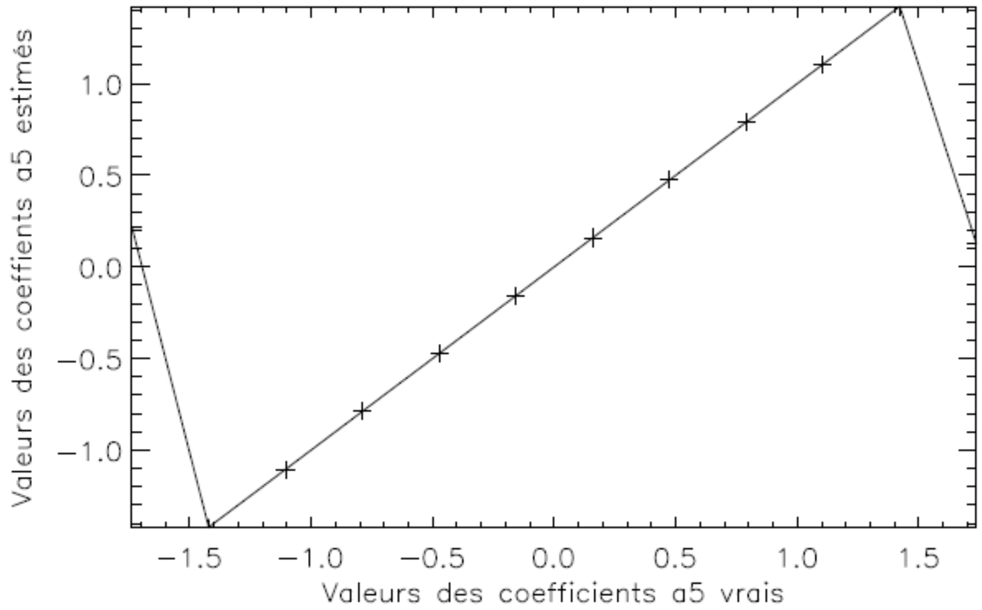
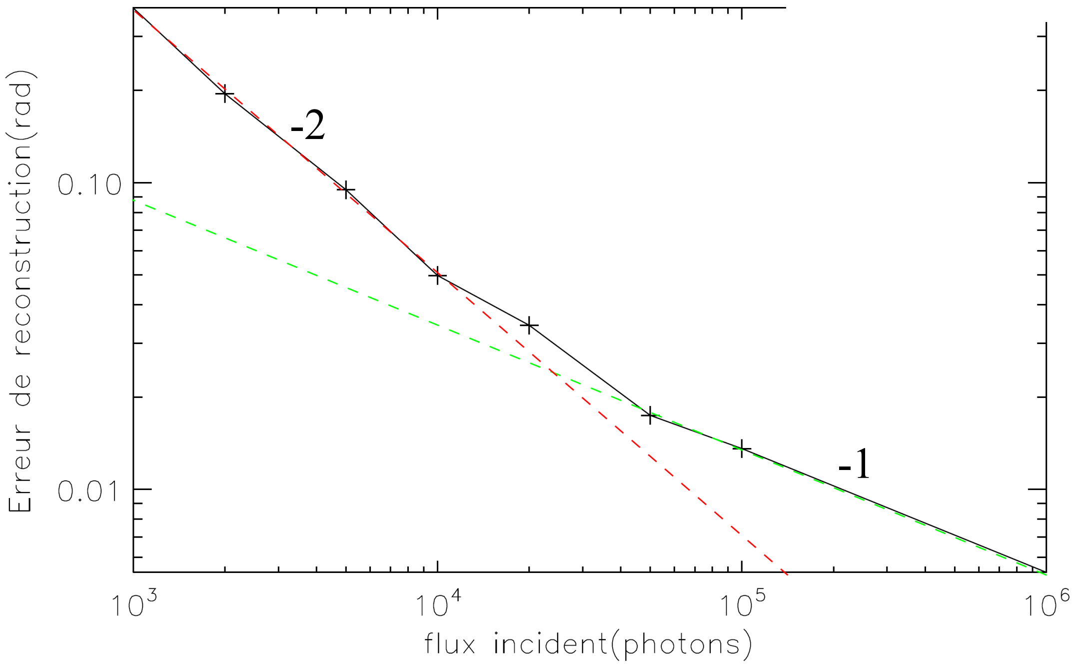
# Characterisation of the coronagraphic WFS

- Linearity




- Error vs. Flux



# Conclusion / perspectives

- Developpement of analytical expression for coronagraphic imaging
  - => validation by simulations
  - Comparison to simple Four Quadrant Phase Mask

- First applications :
  - Simple deconvolution, almost known parameters
  - SPHERE IFS post-processing => see Poster by Marie Ygouf
    - Linearized model applied to myopic coronagraphic deconvolution
  - Coronagraphic Focal plane WFS
    - Exact model applied to static aberrations estimation

  - Simulation performance with real coronagraph
  - Pseudo-Closed-Loop aberrations pre-compensation
  - Experimental validations on AO bench
  - Deal with broadband wavefront sensing => Poster by S. Dandy